Measurement of Thiele Small parameters at Scan-Speak

Measurement of Thiele Small parameters can lead to quite a bit of discussion within the loudspeaker community.

The following is a description of the method used at Scan-Speak.

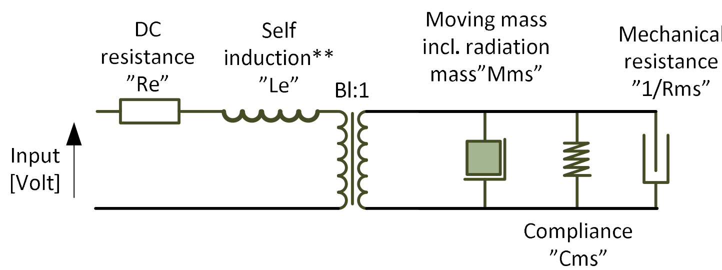
The initial description of measurement and calculation of these Small Signal Parameters comes from Richard Small's papers in the Journal of Audio Engineering in 1972, "Direct Radiator Loudspeaker Analysis". Both Neville Thiele's and Richard Small's papers on this subject dealt with alignment of bass response for loudspeaker systems.

Based on this origin please note that the derivation of T/S parameters are only really meaningful for woofers and in general drivers where an "added mass" or "added compliance" can be used in the measurements. If however a T/S dataset is required for a tweeter or another small tranducer the measurements must be made by e.g. laser or by other means.

It is really important to understand that we are working with "small signal" parameters, which means that the excitation of the moving parts is so small that the mechanical properties can be considered to operate in their linear range. It is also important to realize that any small change of measurement method will result in different results. In fact, there are different ways of measuring these parameters in the market, and they are being used by different systems manufacturers to align or tune loudspeaker systems. So there is no one "correct" set of parameters, but several sets that could potentially be used and the key is to stick with one method and evaluate whether the resulting system response matches your simulation results. If you find an adequate match then that is what you should use.

First of all the Thiele Small parameters relate to the classical "lumped parameter model" of a loudspeaker unit as shown below, where the admittance analogy is used.

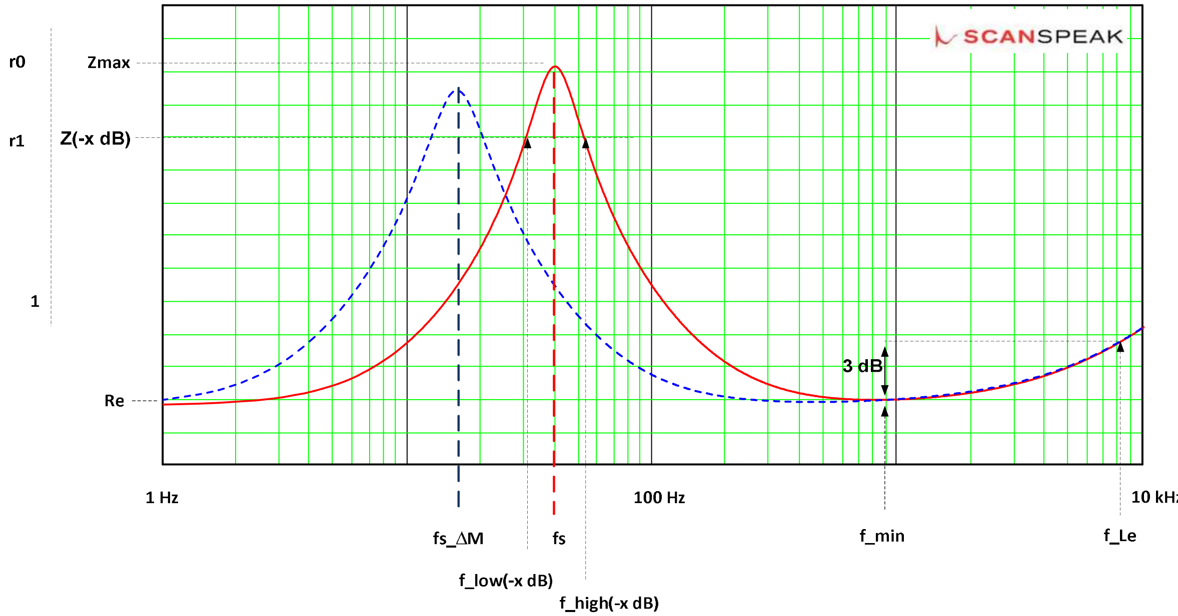
Fig 1. Lumped parameter model



We use the classical, simplified model from fig 1, and the measurement by Scan-Speak's R&D follows Richard Small's derivation of the parameters. We use very low excitation and obtain constant current by using a compressor to maintain a constant voltage over a series resistor. We use pure sine tones from an analog generator and search the resonance frequencies, the side frequencies and the voltages manually.
We use the "added mass" method, and choose an added mass big enough to get a large enough difference in resonance frequency.

Figure 2 illustrates (a bit exaggerated) the principle.

Fig 2. Impedance responses used to derive the parameters



By measurement and calculation we wish to establish values for the following parameters:

Fig 3. List of parameters needed for a full Thiele/Small specification

Re	DC resistance of voice coil	[Ω]
fs	Resonance frequency	[Hz]
Qm	Mechanical Q	
Qe	Electrical Q	
Qt	Total Q	
Mms	Moving mass including the air load/radiation mass	[kg]
Cms	Mechanical compliance	[m/N]
Vas	Equivalent volume	[m^3]
Bl	Force factor	[Tm] or [N/A]
Rms	Mechanical resistance	[Ns/m]
Le	Self induction** of voice coil	[H]

Measurement conditions

Before starting measurement of the parameters, the drive unit must be burned-in by exposing it to a (preferably) sine tone at a frequency slightly below or at the initial resonance frequency and then applying a voltage large enough for the driver to move almost to its maximum excursion. If the driver starts clipping, which is clearly audible, turn back the voltage. The T/S parameters should always be measured on a burned-in driver, because the compliance of a completely new driver will change quite a bit in the first few minutes of excitation. So if the data is to be used for system design, the burned-in dataset is the useful one.

Another thing often overlooked is the conditions under which the driver is measured. Even small drivers will exhibit vibration when exposed to a large enough voltage so it is imperative that the driver is securely clamped during measurement e.g. in a toolshop vice. In order to avoid any influence from physical boundaries make sure that there is room around the measurement setup. Finally, the driver should be positioned vertically so that gravity doesn´t influence the cone area.

We start by recording:

Fig 4. Initial values to be established

Resonance frequency	fs [Hz]
Voltage across the driver at fs	Vmax [Vrms]
Voltage across the series resistance	Vref [Vrms]
Series resistor (known value)	Rref [Ohm]
DC resistance of voice coil	Re [Ohm]
Effective cone area	Sd [m^2]

We can find the value of the constant current $I_{k:}$

$$I_k = \frac{V_{max}}{Z_{max}} = \frac{V_{ref}}{R_{ref}}$$

And thus

$$Z_{max} = R_{ref} \times \frac{V_{max}}{V_{ref}}$$

Determining the Q values

We need to establish the two side frequencies f_low and f_high (fig 4) to calculate the mechanical Q. To facilitate the calculation we normalize values relative to Re=1 and get

$$r_0 = \frac{Z_{max}}{R_e}$$

$$r_1 = \sqrt{r_0} = \sqrt{\frac{R_{ref} \times \frac{V_{max}}{V_{ref}}}{R_e}}$$

We have set the target value where we wish to find the side frequencies as the square root of Zmax/Re. We now calculate the relationship in dB between Zmax and the numerical value of r1 and get a number of dB´s that we need to go below maximum level (see fig. 2).

$$\text{level} = 20 \times \log(\frac{Z_{max}}{R_e \times r_1})$$

If the impedance response around resonance in free air is skewed and asymmetrical due to nonlinearities the square root of Zmax may not be the best value to choose. Another level may be chosen to obtain the best symmetry around the resonance top. If this is the case we must calculate the Qm according to the derivation by R. Small as:

$$Q_m = \frac{f_s}{f_{high} - f_{low}} \times \sqrt{\frac{r_0^2 - r_1^2}{r_1^2 - 1}}$$

Even if we do so, the Q value found will at best have some degree of uncertainty due to the nonlinearities.
For this note and for most drivers, however, we apply the simple expression.

With the sine generator we find the level and record f_low and f_high. The mechanical Q is then:

$$Q_m = \frac{f_s \times r_1}{f_{high} - f_{low}}$$

From which follows

$$Q_e = \frac{Q_m}{r_0 - 1}$$

And finally

$$Q_t = \frac{Q_m \times Q_e}{Q_m + Q_e}$$

Determining the moving mass and the mechanical compliance

We already have the resonance frequency of the driver recorded so we need to add some extra mass to the driver in order to find the resonance frequency with this mass added to the drivers own mass. We use a sticky rubber compound which we weigh out carefully to get the exact mass that we want to add to the driver. The compound is attached symmetrically as close to the driving point of the cone as possible i.e. the voice coil area.
Next we find the resonance frequency and having two resonance frequencies for the same driver and knowing the additional mass makes it possible to calculate the driver´s own mass.

$$M_{ms} = \frac{M_\Delta}{\frac{f_s^2}{f_{\Delta m}^2} - 1}$$

From which we can find

$$C_{ms} = \frac{1 - \frac{f_{\Delta m}^2}{f_s^2}}{4\pi^2 \times f_s^2 \times M_\Delta}$$

And following that:

$$V_{as} = \rho \times c^2 \times Sd^2 \times C_{ms}$$

ρ =1,2 [kg/m³] the density of air (20° C, 50 % RH, 1 atm.)
c =344 [m/s] the speed of sound in air (20° C, 50 % RH)

Calculating the remaining parameters

Calculating the remaining derived parameters using the expression for Qe as a base:

$$Q_e = \frac{2\pi \times f_s \times M_{ms} \times R_e}{Bl^2}$$

Giving:

$$Bl = \sqrt{\frac{2\pi \times f_s \times M_{ms} \times R_e}{Q_e}}$$

Knowing that the maximum impedance at resonance is purely resistive (phase is zero) we also know that

$$Z_{max} = R_e + \frac{Bl^2}{R_{ms}}$$

And thus

$$R_{ms} = \frac{Bl^2}{Z_{max} - R_e}$$

*Self induction***

Finally we need to find a value for Le. In order to do that we seek the frequency of minimum impedance and then the frequency at which the level is 3 dB higher, f_{-3dB}. The expression below is empirically found and is rather simple.

$$L_e = \frac{\frac{R_e \times 20 \times 10^3}{2\pi \times f_{3dB}} + 0{,}5}{20} \times 10^{-3}$$

For many years, other and better models for Le and for the whole electrical impedance have existed and a couple of models have entered high end measurement equipment and are as such applicable. The challenge in using such an enhanced model is the lack of consensus in the industry as one model may suit some customers and another model suits others better. At Scan-Speak, we have been working with a model called “Advanced parameters”, but have stopped including them in our datasheets at this point in time. The jury is currently out on whether Scan-Speak will continue to use a more advanced model and a separate Technical Note about this may emerge in the future.

References.

Richard H. Small, “Direct Radiator Loudspeaker Analysis”, JAES Volume 20 Issue 5 pp. 383-395; June 1972
Knud Thorborg et al. “An Improved Electrical Equivalent Circuit Model for Dynamic Moving Coil Transducers”, Paper 7063, AES 122. conv., Vienna
T/S parameter Measurement procedure at Scan-Speak A/S